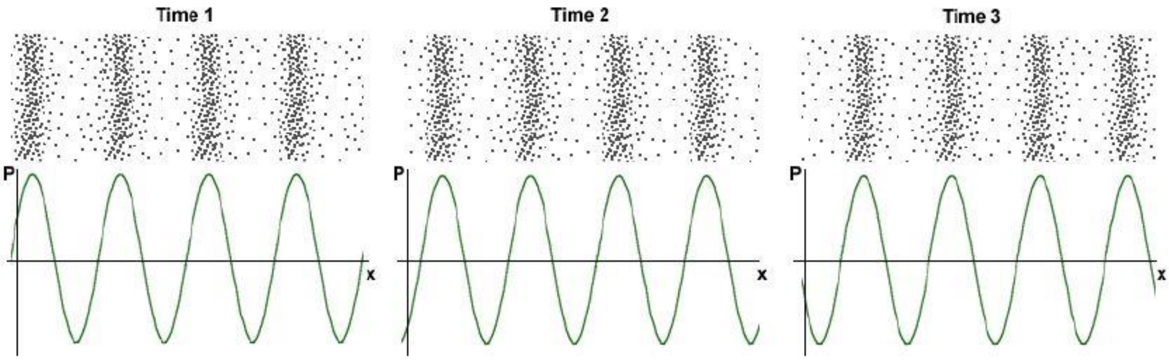
Introduction:

Sound travels through air, a gaseous medium of molecules that are constantly in motion and exerting forces against each other. These forces create air pressure when a large quantity of air molecules are compressed in a small space, air pressure is high, and when air molecules are spread out at a great distance from one another, air pressure is low. Physically, sound is a wave of alternating high and low air pressure that travels through time and space.

A sound wave can be visualized as a chronological sequence of pictures of the compression of air molecules. For a sound wave moving in the rightward direction:



Each individual air molecule moves only a very small distance and does not travel far in the rightward direction, but rather that a rippling effect causes the wave to propagate rightwards.
The graphs of the air pressure at each point in space, where we shift our units so that the horizontal axis indicates normal atmospheric air pressure without the perturbation of sound. The shape of the functions in these graphs is called **the waveform**. The wave is **periodic**, means that it is a repetition of the same pattern across space, and indeed, a musical pitch corresponds to a periodic sound wave. The distance in space between two repetitions of the pattern is called the **wavelength of the wave** or the **period of the waveform**. The number of repetitions of the pattern that move through that point in a single second is called the **frequency of the wave**, measured in Hertz (Hz). For waves moving at the same speed, a longer wavelength implies a longer time for a single repetition of the pattern to move across any given point, and hence it implies a lower frequency.

Frequency X wavelength = speed of wave

Sound travels through the air at about 340 meters per second.

For this particular waveform, known as the sinusoidal wave, there is a clear maximum and minimum amount of air pressure achieved by the wave, and the difference between this and normal atmospheric pressure is called the **amplitude of the wave**.

Speed of Sound - Resonance Tube

Aim: To determine the speed of sound of waves in air.

Apparatus required: glass cylinder, tuning forks, rubber mallet, and meter scale.

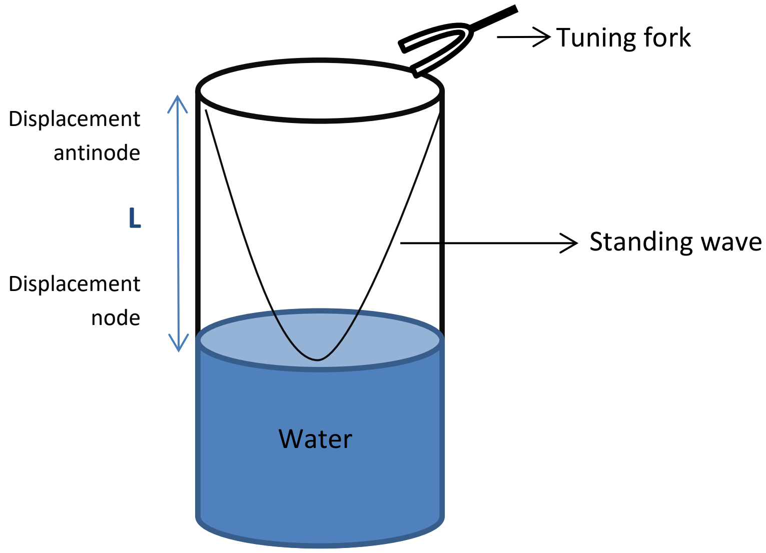
Formula used:

$$\lambda = 4L$$

$$v = \nu\lambda$$

Where λ-wavelength of the standing wave
ν-frequency of the tuning fork
v- Velocity of sound wave
L-length of air column

Theory:
A sound wave is a longitudinal wave in which the wave oscillates along the direction of propagation. For a traveling wave of speed v, frequency f, and wavelength λ, the following relationship holds.
$v = \nu\lambda$
In this experiment, a simple characteristic of the traveling wave , the resonance is used to determine the wavelength (and therefore the speed) of a sound wave.
Consider a sound wave traveling through a resonance tube.



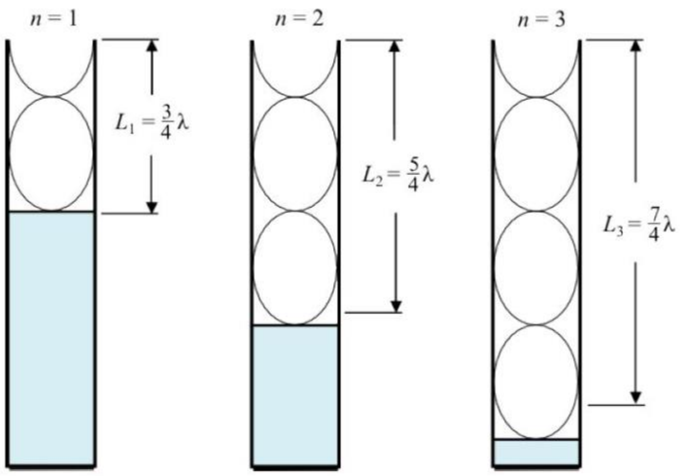
A tuning fork is held by hand just above the open end of the tube. When the tuning fork is struck by a rubber hammer, it vibrates and sound waves are generated. These sound waves travel down the tube and are reflected upon reaching the surface of the water. The incoming and reflected waves interfere and form standing waves. The sound waves reflected from the water surface change their phase by 180^o and therefore are completely out of phase with the incident sound waves. In other words, the amplitude of the standing waves must be zero at the water's surface. This point in space is usually referred to as a node. If a resonance condition is met, the open end of the tube has maximum amplitude of standing sound waves and is called an anti-node.

At constant temperature the speed of sound is fixed; in addition, for a given tuning fork the frequency is also fixed, then accordingly the wavelength of the sound wave should also be fixed. As a result the resonance conditions can only be satisfied when the length of the tube L is such that

$L_n = \frac{1}{4}(2n+1)\lambda$,

Where n = 0, 1, 2, 3, 4... and the length L_n is defined to be the distance measured from the open end of the tube to the water surface. The length of the tube for n=0, is $L = \frac{1}{4}\lambda$.

For higher n,



Procedure :

- Take three tuning forks of known frequencies f.
- Fill the glass cylinder with water to about 10 cm to the open end of the tube. The level of the water in the tube can be adjusted by pouring water in or out.
- Strike the tuning fork with the rubber head of the mallet for forks and place it just above the open end of the tube. Neither the hammer nor the vibrating fork should touch the tube.
- Measure the length of the air column for the water levels where the amplitude of sound is maximum. This length of the air column where the amplitude of sound is maximum is due to resonance.
- Repeat the same for the other tuning fork with a different frequency.

Observations:

Frequency (Hz) (ν)	**Length of air column (cm) (L)**	**Wavelength (cm) λ=4L**	**Velocity of sound (m/s) V=$\lambda\nu$**
341.3	23.2	92.8	316.72
384.0	22.5	90.0	345.60
512.0	15.8	63.2	323.58

$V_{avg} = \frac{316.72+345.60+323.58}{3} = 328.63$ m/s

Result: Therefore the speed of sound was determined using the sound resonance technique and was found to be 328.63 m/s.

Objectives/Goals:

The purpose of this project is to explore the vibrations of a circular membrane when pure sine wave (using a signal generator) is played via a speaker. I wanted to see if the invisible vibrations of the drum could be made visible. I also desired to find out why different frequencies made the membrane vibrate differently

Theory:

Consider a circular drum whose skin has area density (mass per unit area) ρ. If the boundary is under uniform tension T, this ensures that the entire surface is under the same uniform tension. The tension is measured in force per unit distance (newton's per meter).To understand the wave equation in two dimensions, for a membrane such as the surface of a drum, the argument is analogous to the one dimensional case. We parameterize the surface with two variables x and y, and we use z to denote the displacement perpendicular to the surface. Consider a rectangular element of surface of width Δx and length Δy. Then the tension on the left and right sides is TΔy, and the difference in vertical components is approximately

$$(T\Delta y)(\Delta x\frac{\partial^2 z}{\partial x^2})$$

the difference in vertical components between the front and back of the rectangular element is approximately

$$(T\Delta x)(\Delta y\frac{\partial^2 z}{\partial y^2})$$

So the total upward force on the element of surface is approximately

$$(T\Delta x\Delta y)(\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2})$$

The mass of the element of surface is approximately p. Δx. Δy so Newton's second law of motion gives

$$(T\Delta x\Delta y)(\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2})=p\Delta x\Delta y(\frac{\partial^2 z}{\partial t^2})$$

Divide by ΔxΔy and set c = $\sqrt{T/p}$, we get

$$C^2(\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}) = (\frac{\partial^2 z}{\partial t^2})$$

Converting to polar coordinates (r, θ)

$$(\frac{\partial^2 z}{\partial t^2})=c^2(\frac{\partial^2 z}{\partial r^2}+\frac{1}{r}\frac{\partial z}{\partial r}+\frac{1}{r^2}\frac{\partial^2 z}{\partial \theta^2})$$

Let the separable solutions of this equation be of the form

Z=f(r)g(θ)h(t)

Substituting this into the wave equation and Dividing by f(r)g(θ)h(t) gives

$$\frac{h''(t)}{h(t)}=c^2(\frac{f''(r)}{f(r)}+\frac{1}{r}\frac{f'(r)}{f(r)}+\frac{1}{r^2}\frac{g''(\theta)}{g(\theta)})$$

In this equation, the left hand side only depends on t, and is independent of r and θ, while the right hand side only depends on r and θ, and is independent of t. Since t, r and θ are three independent variables, this implies that the common value of the two sides is independent of t, r and θ, so that it has to be a constant. Let this constant be a negative real number, $-\omega^2$. So we obtain two equations,

$$h''(t) = c^2 h(t)$$

$$\frac{f''(r)}{f(r)} + \frac{1}{r}\frac{f'(r)}{f(r)} + \frac{1}{r^2}\frac{g''(\theta)}{g(\theta)}) = \frac{-\omega^2}{c^2}$$

The general solution to equation is a multiple of the solution

h(t)=sin(wt+ϕ)

where ϕ is a constant determined by the initial temporal phase. Multiplying equation by r^2 and rearranging, we obtain

$$r^2\frac{f''(r)}{f(r)} + r\frac{f'(r)}{f(r)} + \frac{\omega^2}{c^2} = -\frac{g''(\theta)}{g(\theta)}$$

The left hand side depends only on r, while the right hand side depends only on θ, so their common value is again a constant. This makes g(θ) either a sine function or an exponential function, depending on the sign of the constant. But the function g(θ) has to be periodic of period 2 π since it is a function

of angle. So the common value of the constant must be the square of an integer n, so that

$$g''(\theta) = -n^2 g(\theta)$$

and g(θ) is a multiple of sin(n θ + ϕ). Here, ϕ is another constant representing spatial phase.

So we obtain

$$r^2\frac{f''(r)}{f(r)} + r\frac{f'(r)}{f(r)} + \frac{\omega^2}{c^2} = n^2$$

Multiplying by f(r), dividing by r^2 and rearranging, this becomes

$f''(r) + \frac{1}{r}f'(r) + (\frac{\omega^2}{c^2} - \frac{n^2}{r^2})$f(r)=0

the general solution to this equation is a linear combination of J_n(ω r/c) and Y_n(ω r/c). But the function Yn(ω r/c) tends to $-\infty$ as r tends to zero, so this would introduce a singularity at the centre of the membrane. So the only physically relevant solutions to the above equation are multiples of J_n(ω r/c).

$$\boxed{z = A\, J_n(\omega r/c)\sin(\omega t + \phi)\sin(n\theta + \phi)}$$

are solutions to the wave equation.

If the radius of the drum is a, then the boundary condition which we must satisfy is that z = 0 when r = a, for all values of t and θ. So it follows that $J_n(\omega a/c) = 0$. This is a constraint on the value of ω. The function J_n takes the value zero for a discrete infinite set of values of its argument. So ω is also constrained to an infinite discrete set of values.

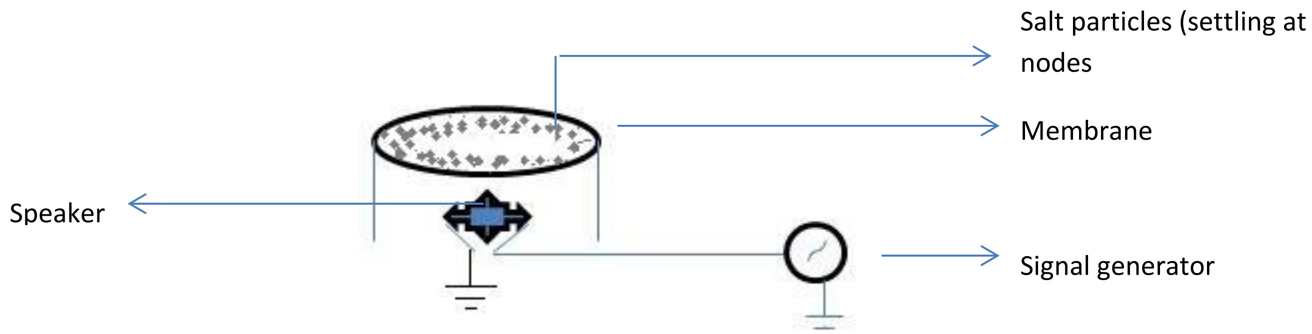
We have seen that to choose a vibrational mode, we must choose a nonnegative integer n and we must choose a zero of $J_n(z)$. Denoting the kth zero of J_n by $j_{n.k}$, the corresponding vibrational mode has frequency $(cj_{n,k}/2\pi a)$, which is $j_{n,k}/j_{0,1}$ times the fundamental frequency.

In the late eighteenth century, Chladni discovered a way to see normal modes of vibration. He was interested in the vibration of plates, but the same technique can be used for drums and other instruments. He placed sand on the plate and then set it vibrating in one of its normal modes, using a violin bow. The sand collects on the stationary lines .

In practice, for a drum in which the air is confined the fundamental mode of the drum is heavily damped, because it involves compression and expansion of the air enclosed in the drum. So what is heard as the fundamental is really the mode with n = 1, k = 1. The higher modes mostly involve moving the air from side to side. The inertia of the air has the effect of raising the frequency of the modes with n = 0, especially the fundamental, while the modes with n > 0 are lowered in frequency in such a way as to widen the frequency gaps. For an open drum, on the other hand, all the vibrational frequencies are lowered by the inertia of the air, but the ones of lower frequency are lowered the most.

Methods/Materials: Speaker, signal generator, A hollow cylindrical stand to mount the membrane, A drum membrane.

Circuit diagram:

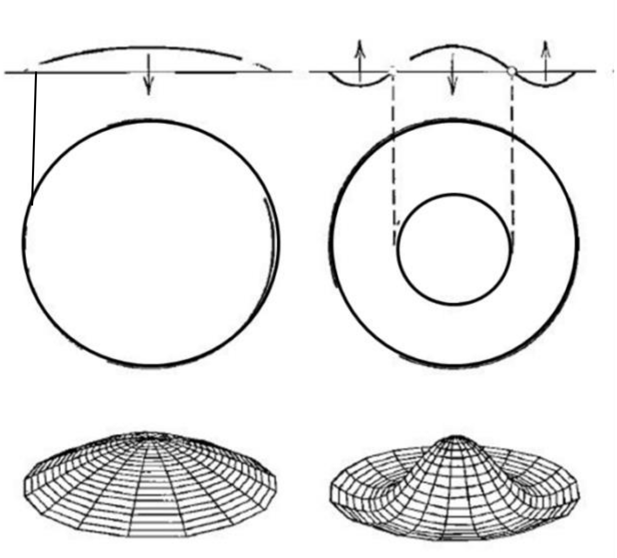


Procedure:

- The drum membrane was placed on the hollow cylindrical stand and the speaker was placed under the membrane.
- The speaker was connected to the signal generator (in sine mode)as shown in the figure.
- Fine salt was sprinkled on the membrane.
- As the frequency was changed the patterns formed on the membrane were observed.

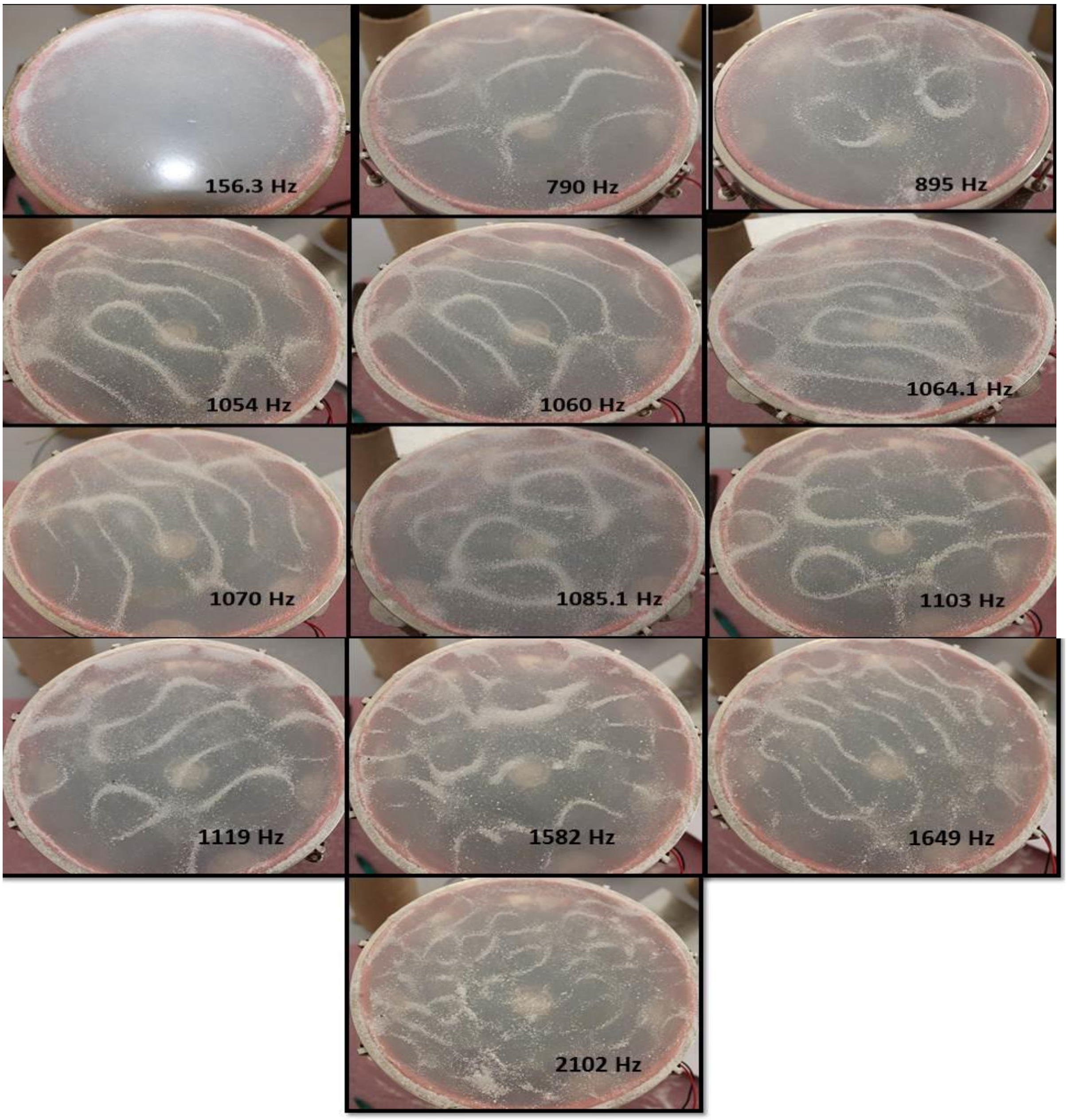
Conclusions/Discussion:

- Evidence of the different modes of vibrations of the membrane with the change in frequency was explored by observing the patterns formed by fine salt sprinkled over the membrane.
- As the frequency was changed, the patterns also changed and became more and more complex as frequency increased.
- It was also seen that at certain values of frequencies the membrane vibrations were very strong. These frequencies correspond to the resonant frequencies of the membrane.
- At the resonant frequencies, a tap on the membrane gave the sound of the musical note corresponding to that very frequency.
- On solving the equation for the wave equation it is found the the fundamental mode has the whole membrane vibrating . The stationary points has the following picture



<u>n=1 , k=1</u>

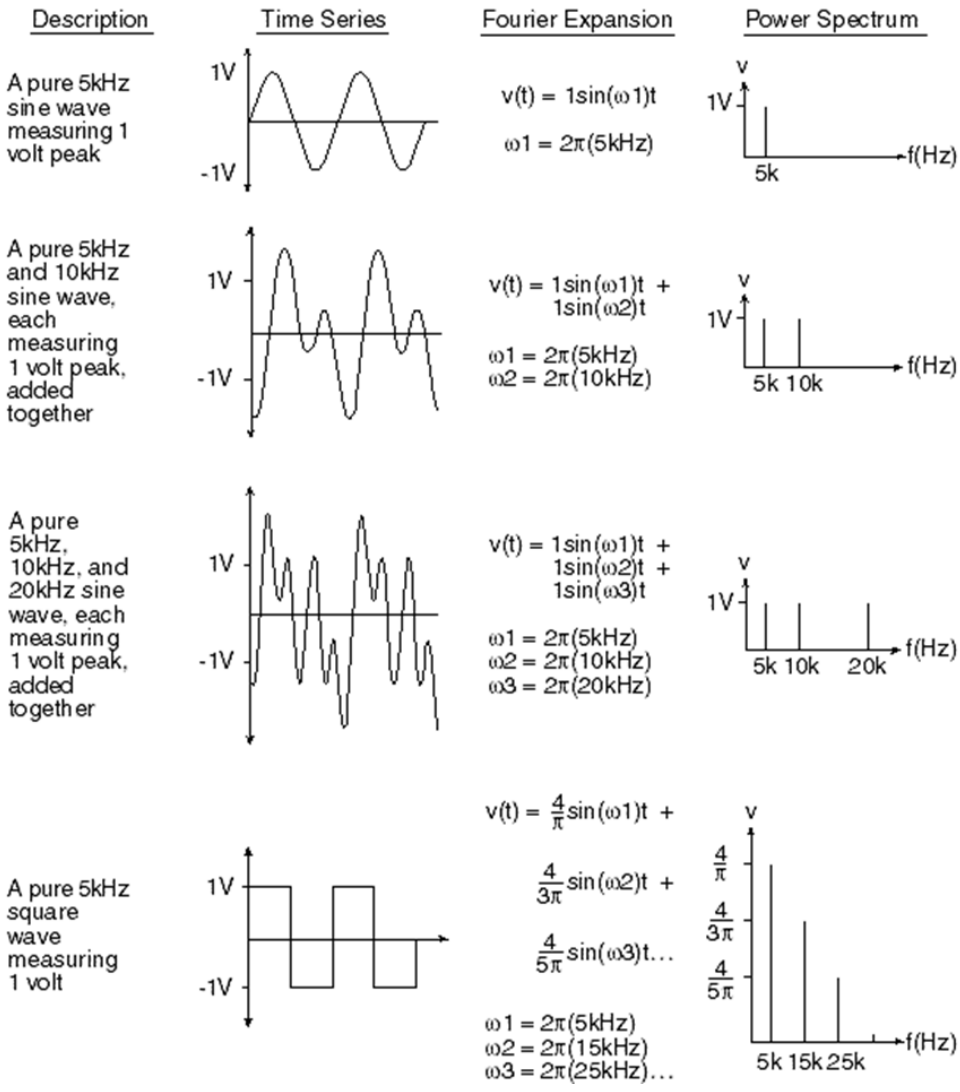
THE PATTERNS OBSERVED FOR DIFFERENT FREQUENCIES



Aim: To understand why different instruments have distinctive timbre though they play the same note.

Theory:

Fast Fourier Transform (FFT): The Fourier transform converts waveform data in the time domain into the frequency domain. The Fourier transform accomplishes this by breaking down the original time-based waveform into a series of sinusoidal terms, each with a unique magnitude, frequency, and phase. This process, in effect, converts a waveform in the time domain that is difficult to describe mathematically into a more manageable series of sinusoidal functions that when added together, exactly reproduce the original waveform. Plotting the amplitude of each sinusoidal term versus its frequency creates a power spectrum, which is the response of the original waveform in the frequency domain.



One method was developed in 1965 by James W. Cooley and John W. Tukey1 Their work led to the development of a program known as the fast Fourier transform. The fast Fourier transform (FFT) is a computationally efficient method of generating a Fourier transform. The main advantage of an FFT is speed, which it gets by decreasing the number of calculations needed to analyze a waveform.

The FFT algorithm reduces an n-point Fourier transform to about

(n/2) log2 (n)

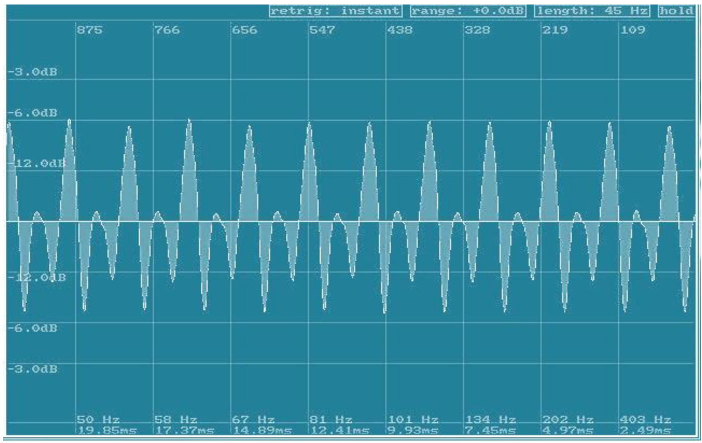
complex multiplications. But the increase in speed comes at the cost of versatility. The FFT function automatically places some restrictions on the time series to be evaluated in order to generate a meaningful, accurate frequency response. Because the FFT function uses a base 2 logarithm by definition, it requires that the range or length of the time series to be evaluated contains a total number of data points precisely equal to a 2-to-the-nth-power number (e.g., 512, 1024, 2048, etc.). Therefore, with an FFT you can only evaluate a fixed length waveform containing 512 points, or 1024 points, or 2048 points, etc.

MUSICAL INSTRUMENTS

As we investigate musical instruments, we will discover that some type of vibrating system produces all musical sounds. The strings on the guitar, or the air column in the clarinet, and the head of the drum are examples of vibrating systems. The vibrating systems on most musical instrument are made up of two or more vibrating systems working together to produce sounds loud enough to be heard by the human ear.

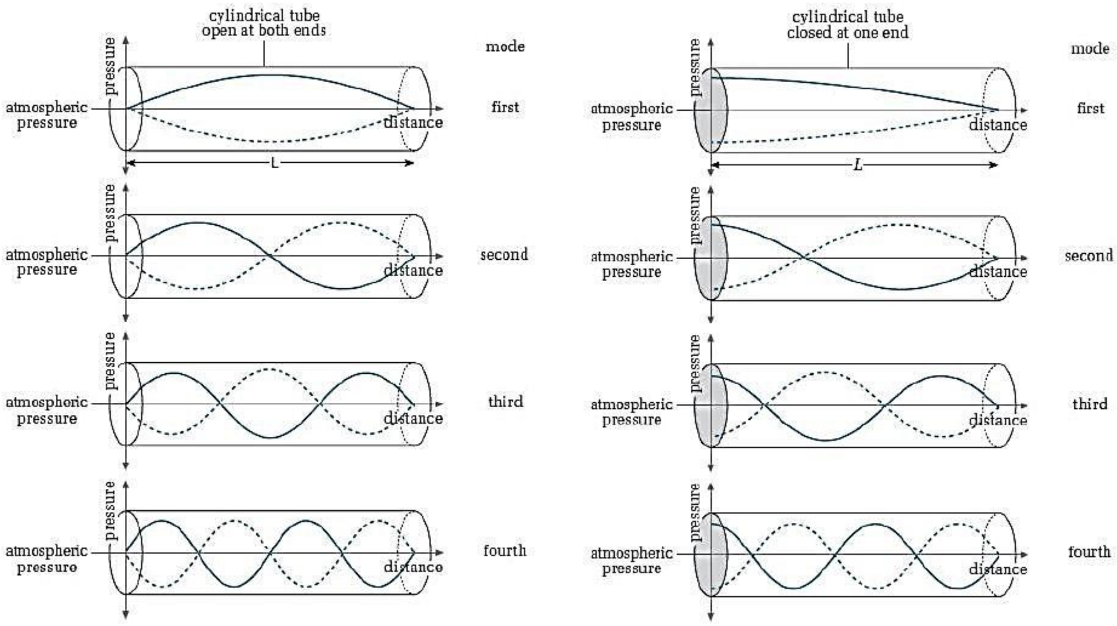
Wind instruments depend on the vibration of a column of air to produce sound. The column of air vibrates when wind is blown into or across an instrument. There are two types of wind instruments, brass and woodwind instruments. Brass instruments are played by vibrating the lips and pressing them against the mouthpiece of the instrument. This causes the air column to vibrate and create sound. Woodwind instruments such as the clarinet need a reed to make the air columns vibrate. The column of air vibrates in the flute and piccolo when air is blown across a hole. Higher or lower pitch can be produced in these instruments by making the air column shorter or longer.

Suppose that a saxophone and a flute play the same pitch at the same volume level, It is easy to hear the differences between these sounds. But even though our brain perceives the sinusoidal wave as a pure musical pitch, most musical sounds that we hear are not pure pitches. Graphs of the waveform for a trumpet is shown below:



It is observed that waveform is periodic with the same period, but the shape of the waveforms is different, and is not purely sinusoidal. Though each wave corresponds to a pure sinusoidal wave of frequency f, there are various waves of higher frequency added to it.

Consider the instruments of the woodwind family, such as flutes, saxophones,clarinet as well as the brass family, such as trumpets, trombones. The model of the wind instrument can be approximated for the vocal tract of the human voice. Sound in these instruments is shaped by the reverberation of sound waves within a tube, which can be modeled as a cylinder with open or closed ends. Again, the tube allows for modes of oscillation corresponding to standing sound waves inside the tube of varying frequencies. An open end of the tube must correspond to a point of zero pressure difference from normal air pressure, similar to how a fixed end of a string cannot be displaced. If the tube has a closed end, the endpoint of the tube must be a point of maximal pressure difference from normal air pressure. (This is due to the condition that the displacement of the air molecules at the closed end must be fixed at zero and to a relationship between the pressure and displacement.) Hence, the modes of oscillation for tubes with two open ends, such as the flute, or tubes with one closed end, such as the saxophones, are those shown below:



In the case of two open ends, the wavelength of the first mode of oscillation is twice the length of the tube, and the frequency of the nth mode is n times the frequency of the first mode. In the case of one closed end, the wavelength of the first mode of oscillation is four times the length of the tube, and the frequency of the nth mode is 2n-1 times the frequency of the first mode.

The frequency of the first mode of oscillation for a string or tube is known as its fundamental frequency, or the first harmonic, and the frequency that is n times the fundamental frequency is the nth harmonic.

Apparatus required: pick up , software to give FFT, instrument (saxophone, flute, trumpet, clarinet) and instrument players.

Software used: REAPER-

Procedure:

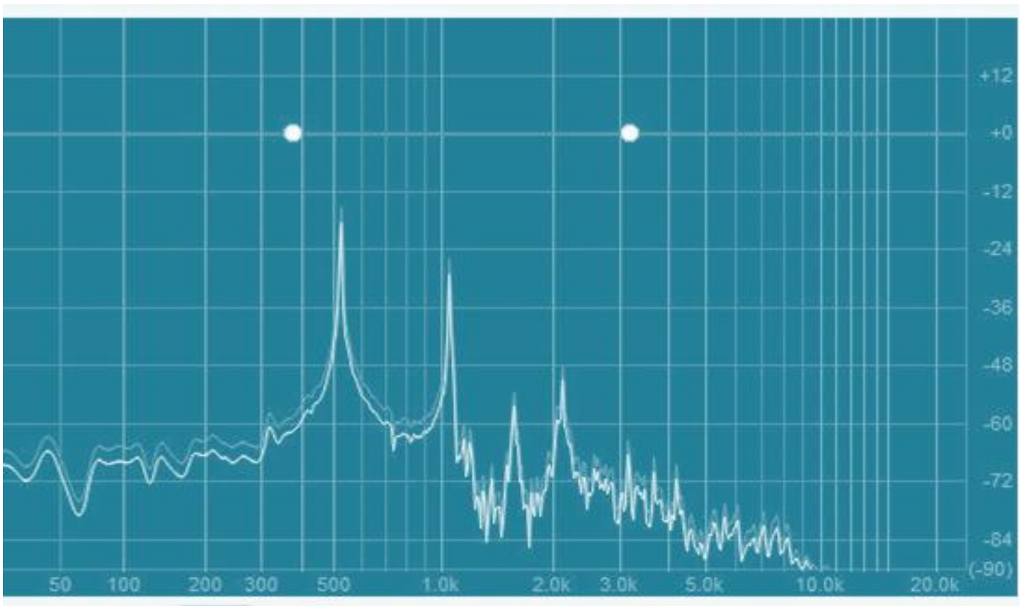
- First samples of the C4 note were recorded for different instruments (saxophone, flute, clarinet, trumpet) using a pickup onto different tracks.
- Then FFT plugin was applied to the tracks individually by muting the other tracks.
- The FFT patterns were studied.
- Also a comparative study for different instruments was carried out by plotting relative amplitude versus the frequency graph. The relative amplitudes were calculated using the amplitude for C5 note as reference 0.

Observations:

Note: C5

Instrument: Flute

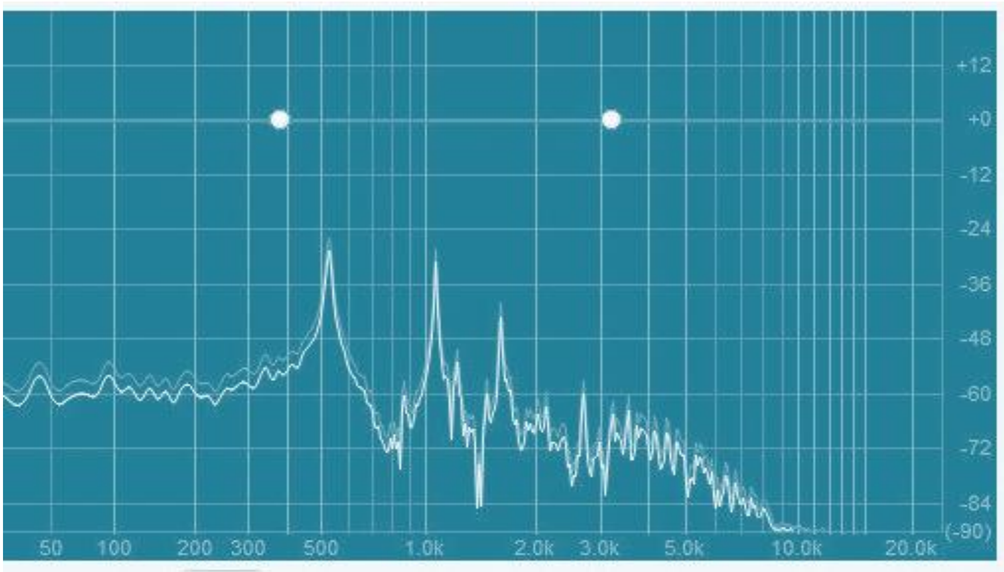
f (Hz)	Note	A (dB)
21	**C5**	-15.09
785	**G5**	-63.45
1048	**C6**	-25.26
1582	**G6**	-48.88
2080	**C7**	-44.81
2625	**E7**	-60.69
3187	**G7**	-63.94
3685	A#7	-66.39
4265	**C8**	-72.09



Note: C5

Instrument: Clarinet

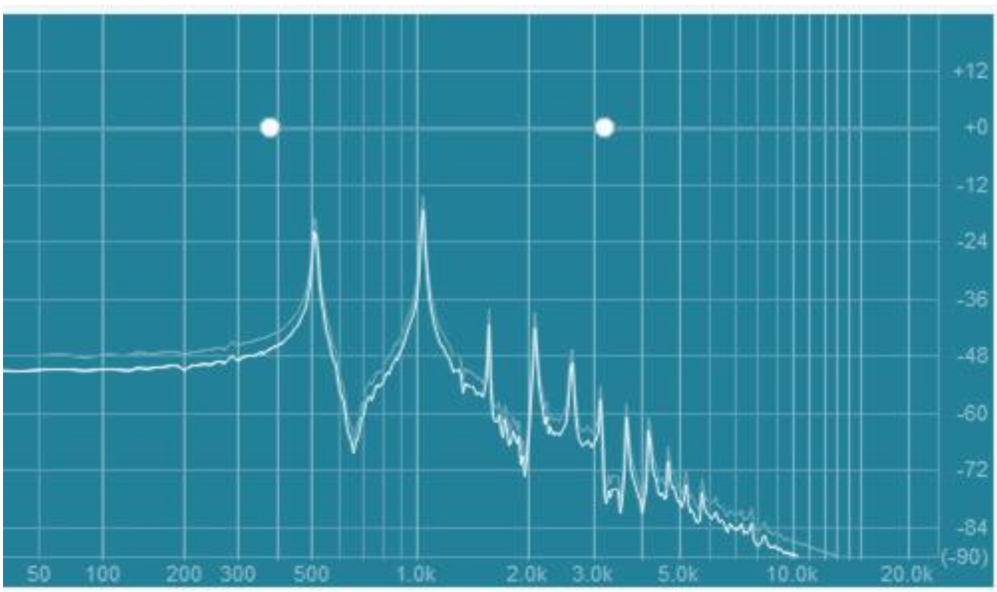
f (Hz)	Note	A (dB)
527	**C5**	-24.04
785	**G5**	-66.39
1048	**C6**	-27.71
1330	**E6**	-63.13
1605	**G6**	-39.11
1839	A#6	-61.09
2103	**C7**	-59.46
2291	D7	-66.39
2689	**E7**	-56.61
2958	F#7	-65.98
3228	G#7	-61.09
3556	A7	-59.46



Note: C4

Instrument: Trumpet

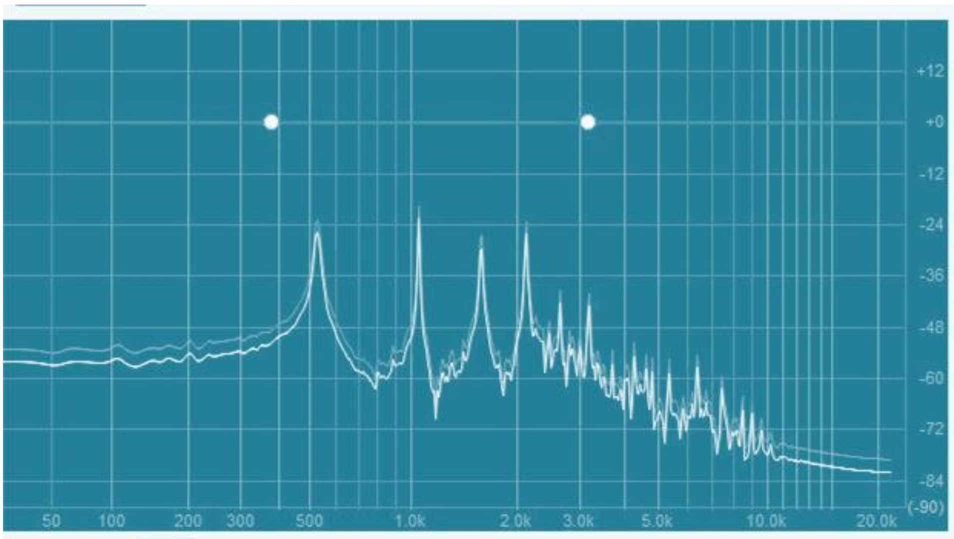
f (Hz)	Note	A (dB)
521	**C5**	-17.53
1025	**C6**	-14.68
1564	**G6**	-36.66
2080	**C7**	-38.29
2589	**E7**	-45.68
3111	**G7**	-53.76
3644	A#7	-57.43
4160	**C8**	-60.28
4693	D8	-66.39
5232	**E8**	-71.27
5765	F#8	-73.31



Note: C5

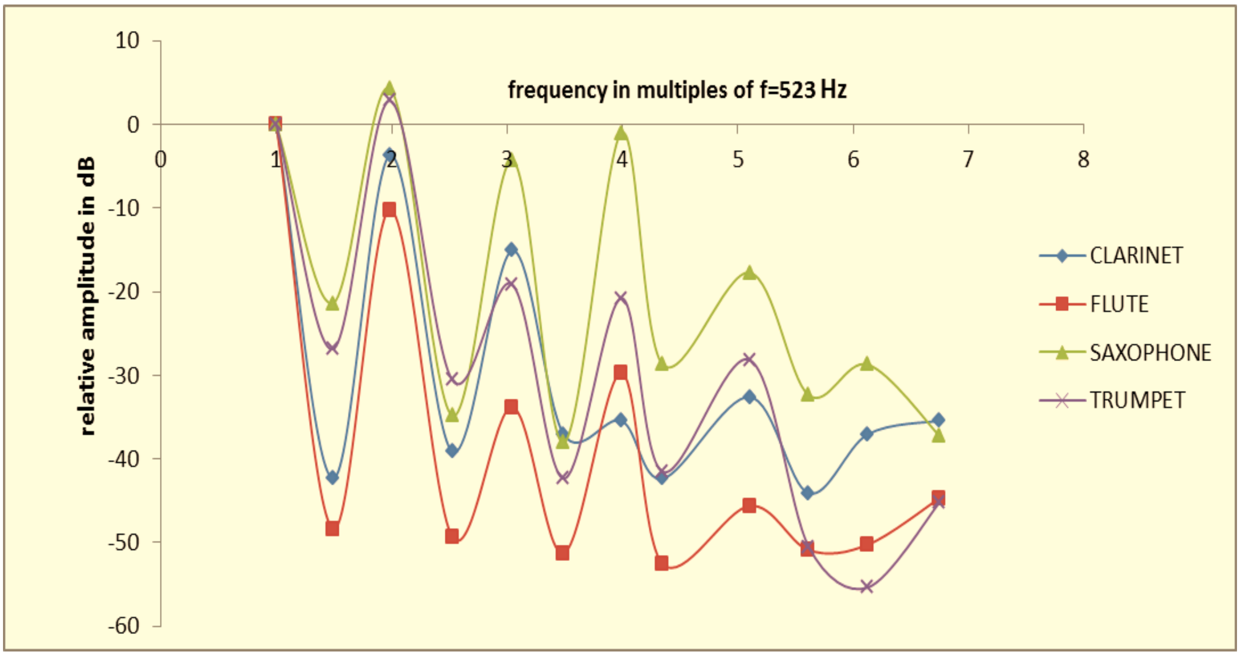
Instrument: saxophone

f (Hz)	Note	A (dB)
521	**C5**	-23.03
1060	**C6**	-18.75
1582	**G6**	-27.3
2103	**C7**	-24.04
2654	**E7**	-40.74
3187	G7	-40.33
3386	G#7	-51.73
3685	A#7	-52.95
6421	G8	-58.24
6990	A8	-57.02
7511	A#8	-58.24

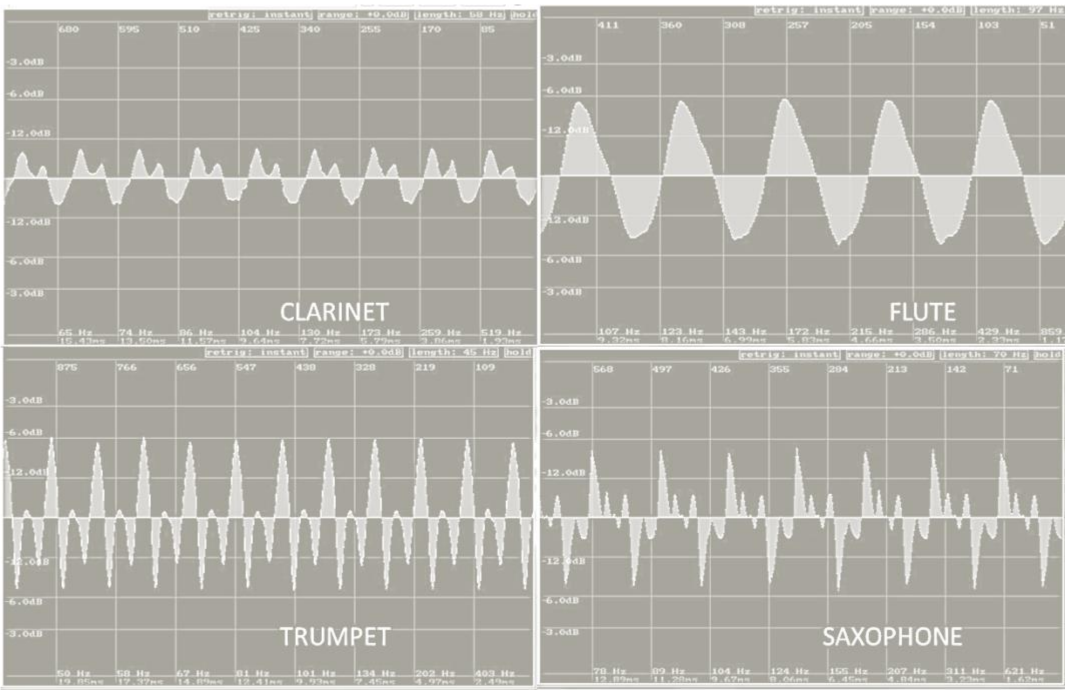


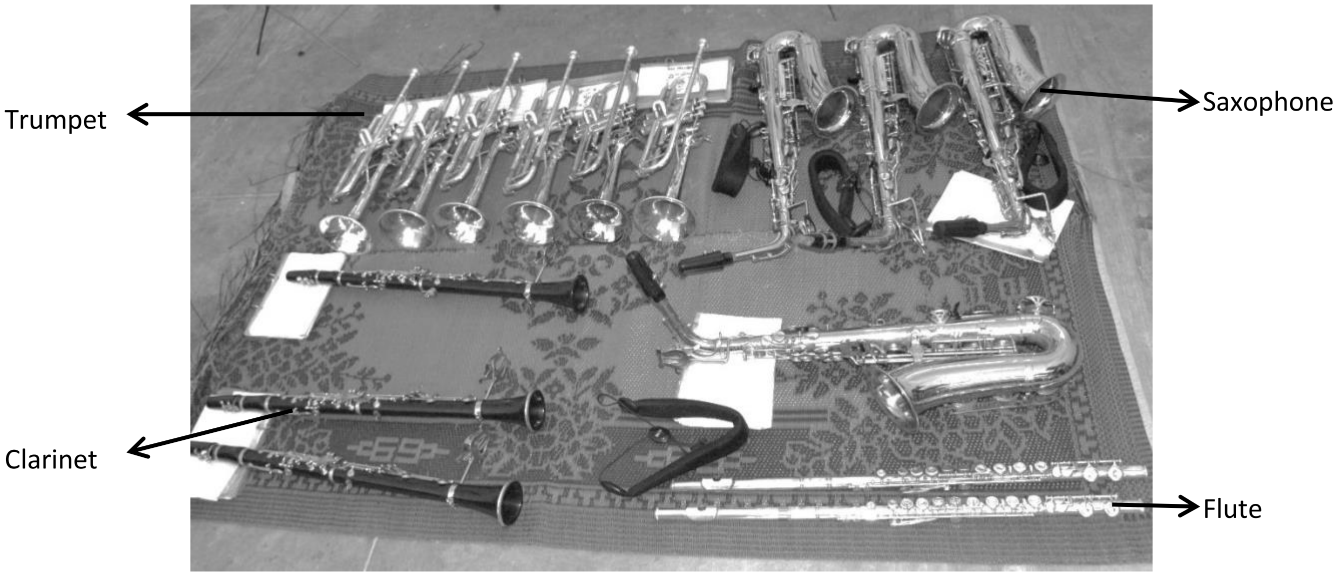
Comparison graph for amplitude of overtones in different notes

Frequency (Hz)	Approx. Relative frequency f=523hz	Clarinet	Clarinet Relative amplitude	Flute	Flute Relative amplitude	Saxophone	Saxophone Relative amplitude	Trumpet	Trumpet Relative amplitude
527	1	-24.04	0	-15.09	0	-23.03	0	-17.53	0
785	1.5	-66.39	-42.35	-63.45	-48.36	-44.4	-21.37	-44.4	-26.87
1048	2	-27.71	-3.67	-25.29	-10.2	-18.75	4.28	-14.68	2.85
1330	2.5	-63.13	-39.09	-64.35	-49.26	-57.84	-34.81	-48.06	-30.53
1605	3	-39.11	-15.07	-48.88	-33.79	-27.3	-4.27	-36.66	-19.13
1839	3.5	-61.09	-37.05	-66.39	-51.3	-61.09	-38.06	-59.87	-42.34
2103	4	-59.46	-35.42	-44.81	-29.72	-24.04	-1.01	-38.29	-20.76
2291	4.5	-66.39	-42.35	-67.64	-52.55	-51.73	-28.7	-59.06	-41.53
2689	5	-56.61	-32.57	-60.69	-45.6	-40.74	-17.71	-45.685	-28.155
2958	5.5	-68.13	-44.09	-65.98	-50.89	-55.39	-32.36	-68.01	-50.48
3228	6	-61.09	-37.05	-65.34	-50.25	-51.73	-28.7	-72.9	-55.37
3556	6.5	-59.46	-35.42	-59.76	-44.67	-60.28	-37.25	-62.72	-45.19



Waveforms for the instruments derived from oscilloscope:





Conclusions:

- A flute has the least amplitude for higher overtones compared to all the other three instruments. Also the waveform is almost pure in nature. This can be attributed to the shape of the instrument. Flute is approximately a perfect open ended cylindrical pipe unlike the other instruments which have varied shapes. The most salient feature of the flute is the limited number of harmonics with an intensity that decreases monotonously.

- The saxophone is a wind instrument implying a limited number of harmonics, and spectra should mainly compose of odd harmonics because of the reed. But a short glance at figures shows that both predictions are wrong. The even harmonics are as strong as the odd. The sound remains very rich in harmonics even in the higher frequencies, far more than for the clarinet, and the timbre changes only slightly between the first and the second frequency. The saxophone does not behave at all like a clarinet. The main reason is the form of the bore: in a cone, the standing waves are not plane but spherical .This has profound implications for the standing wave pattern. A conical bore shows therefore a complete harmonic series, be it excited with a reed, the lips or an edge. It would seem also that the conical pipe of the saxophone favors the higher harmonics as compared to the cylindrical bore of the clarinet.
- The clarinet shows a fascinating behavior: though it is a pipe closed at one end, but the sound it gives has strong even harmonics. But it also known that the timbre varies strongly as the pitch is increased, with a very distinctive sound for each register. This is due to several facts. First, the bore of the clarinet is not perfectly cylindrical but has tapered and slightly conical sections. Second, the flared bell, the constricting mouthpiece and the tone holes (even if they are closed) perturb significantly the standing waves.
  It is also interesting to note that clarinet actually behaves like a closed pipe having predominant odd harmonics at lower registers.

- Trumpet belongs to the family of brass instruments. It has only three keys but it is the lip movement of the player that determines the pitch of the note played. It can be observed that higher overtones become less predominant. The fifth overtone is itself 30 dB lesser than the fundamental.